The Problem Statement

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The first step of problem solving

Definition of a well-defined problem: The ideal USIT problem statement has a minimum set of objects harboring a single unwanted effect and provides root causes based on attributes of the objects.

Of course, this is the ideal. Few real world problems are presented so well defined. Thus, the problem solver begins with incomplete, convoluted, or simply misunderstood information. From this must be extracted the problem or problems to be addressed.

The structure of USIT gives guidance, a target (the definition of a well-defined problem), and tools with which to construct efficiently a viable problem statement. It also admonishes one to select words carefully and avoid psychological barriers presented by object names, for example. One of the devices for circumventing such barriers is the use of sketches to bring out the critical features of the objects in the problem. This exercise effects agreement between one's abstract and concrete perceptions of the problem.

Every problem solving exercise begins with construction of a problem statement or interpretation of a given statement. This is a period of mental wrestling with words and images to establish the starting point of subsequent problem analysis. It can't be overstressed how important is this mental exercise. Every effort should be made to achieve a well-defined problem statement, both verbal and graphic. The clarity of thinking achieved in this exercise forms the basis of ensuing analysis and solution generation phases. It usually happens that even in this phase new solution concepts arise.

Experiences with problem statements

I recently received by email two interesting problems to solve. Although I did not solve them using USIT, my experience with them reminded me of the importance of clear thinking in reading and constructing problem statements.

The first problem was stated as follows:

"What are the chances of flipping a penny four times and getting at least two tails?"

The problem appeared to be well defined and even straightforward to solve. In a few minutes I had the answer of 3/8. It was then pointed out to me that the answer is actually much larger than this (by more than 80%). I reread the problem statement more closely and discovered that I had overlooked the conditional "... at least ...".

Reflecting on this experience, I decided that because I could see how to solve the problem, my eagerness to get it done blinded my reading ability. So, when the second problem arrived, I was prepared to be more cautious.

The second problem:

"Below are five consecutive numbers that are the same upside-down as they are right side up. What is the next four-digit number in this sequence?

1961, 6009, 6119, 6699, 6969 ?"

My first reaction was that the statement was incorrect. When I turned the numbers upside down I got 1691, 9006, 9116, 9966, and 9696. Since none of these read the same as the originals, it was evident that I had misinterpreted the problem. Then it occurred to me that I had not turned the numbers "upside-down" as intended. If, of course, the numbers are rotated 180^{o} about an axis perpendicular to the page they do read the same.

Reflecting on this observation led me to wonder how I had misinterpreted the intended rotation. Evidently the phrase "... the same upside down ..." caused me to mentally rotate the numbers 180^{o} about a horizontal axis in the plane of the page. Apparently I unconsciously view the latter rotation as a summersault and the former as a handspring.

This observation led me to wonder how the manner of rotation affects the original problem. In this problem, rotation needs to produce readable digits, not necessarily the same digits. Two of three orthogonal axes have

been referred to here: one perpendicular to the page and one horizontal in the page. The other is vertical in the page. These three axes seem appropriate for examining the issue of rotating numbers.

Of the ten digits - 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 - their possible 180^{o} rotations into readable orientations (not necessarily identical orientations) are as follows:

Rotation about an axis perpendicular to the page ... 0, 1, 6, 8, 9

Rotation about a horizontal axis in the page ... 0, 1, 3, 8, 9

Rotation about a vertical axis in the page ... 0, 1, 8

Clearly, there is opportunity for modified problem statements yielding different answers.

Deja vu

We all have experienced these kinds of misinterpretations of problems throughout our academic careers. Our instructor has one view of a problem while we have another. Both look at the same words and illustrations but interpret them differently. Would that these experiences could become ingrained in our problem-solving psyches as we attack new problems.